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# RATIONAL FUNCTIONS

**MATH AND SCIENCE TUTORING**
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## Rational Expressions

*The term "rational" comes from ratio; a rational expression is a ratio of expressions.*

*A rational number is a number formed by the division, or "ratio," of two integers P and Q, where Q ≠ 0, and is characterized by a decimal representation that either terminates or repeats.*

Examples

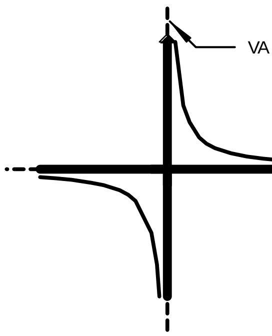
$$\frac{P}{Q} = \frac{1}{4} = 0.25 \qquad \frac{P}{Q} = \frac{1}{3} = 0.\overline{333}$$

*A rational function is a function formed by the division, or "ratio," of two polynomials P(x) and Q(x) where Q(x) ≠ 0, and is characterized by a discontinuity at Q(x) = 0.*

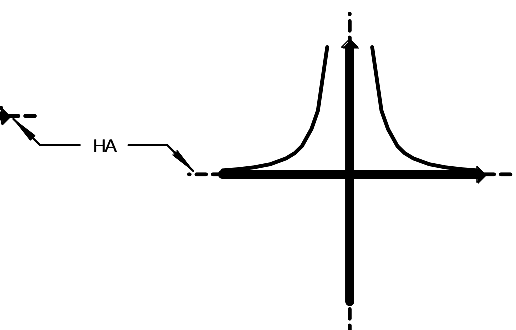
Example

$$\frac{P(x)}{Q(x)} = \frac{x^4 - 3x^3}{x^3 - 3x^2 - x + 3}$$

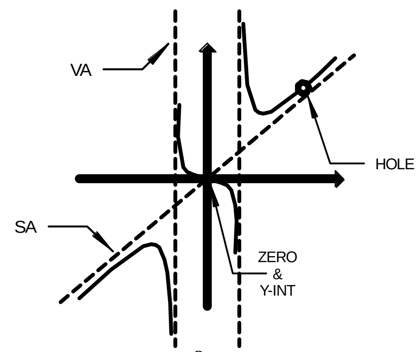
## Parent Functions




$$y = \frac{1}{x}$$




$$y = \frac{1}{x^2}$$




$$y = \frac{x^3(x-3)}{(x^2-1)(x-3)}$$

**Compound function**

**VA: Vertical Asymptote ∞ HA: Horizontal Asymptote ∞ SA: Slant Asymptote ∞ Y-Int: Y-Intercept ∞ Zeros ∞ Holes**

All rational functions have some of these distinguishing features depending on the properties of the functions composing the numerator and denominator. Graphs can cross horizontal and slant asymptotes but never cross a vertical asymptote.

## Finding Critical Points

For a rational function with polynomials $P(x)$, for a numerator, and $Q(x)$, for a denominator, with no common factors, leading coefficients $p$ and $q$, and degrees $m$ and $n$, respectively:

**Zeros**
$P(x) = 0$

**Vertical Asymptotes**
$Q(x) = 0$

**Y-Intercepts**
$x = 0$

**Holes**
...at factors cancelled during simplification process, i.e., $(x - 3)$ above.

**Horizontal Asymptotes**
$m < n$: $y = 0$
$m = n$: $p/q$
$m > n$: none

**Slant Asymptotes**
$m = n + 1$ *and* $P(x)/Q(x)$ has a remainder; SA at $ax + b$ part of quotient (by long division).

## Other Tests

**Sign Test**

| | a | | b | | c | |
|---|---|---|---|---|---|---|
| (x - a) | - | | + | | + | + |
| (x - b) | - | | - | | + | + |
| (x - c) | - | | - | | - | + |
| Product | - | | + | | - | + |

**T-Table**

| x | y |
|---|---|
| x | f(x) |
| 0 | y-int |
| x | zeros |
| x | holes |
| VA | asymp |
| asymp | HA |

**End-Point Behavior**

How does the parent function of the simplified expression behave as $x \to \pm\infty$?

**Bounces and Jogs**

Multiplicity:
EVEN multiples of a factor "bounce" at their zero;
ODD multiples "jog" through their zero, i.e., $x^3$.

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