A periodic table of dimensions

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Abstract

A dimensional space with exponentially scaled axes representing time t, space d, mass m, and electric charge q is populated with over 900 physical quantities found in the literature. Cataloging these quantities by their dimensionality produces a 4D periodic table of 105 unique physical dimensions. The table reveals the organization of the dimensions, and helps clarify some nomenclature. The value of dimensional constants become a function of the plank units in this space. Groupings, trends, and properties of the dimensions, as well as some "missing" dimensions are discussed. Suggested student exercises include a calculus based derivation of an equation of motion out to the 6th time-derivative of motion, including jerk, snap, crackle, and pop, and a dimensional analysis lab. Treating the (t, d, m, q) quadruplets of each dimension as position vectors reveals natural law as linear relations among the dimensions. The use of the table and its accompanying *Catalog of Synonymous Dimensions* for modeling complex physical phenomena is also examined.

I. INTRODUCTION

Ask any chemistry teacher if they would prefer to teach chemistry with, or without, the periodic table of elements and you would be hard pressed to find even one who would prefer to teach chemistry without it. What if we had something like that for physics? What would the elements be? How would those elements be arranged? How could it be used?

While the concept of a periodic arrangement was certainly influenced by Mendeleev, the foundations presented here begin with Minkowski. In introducing the concept of a worldline in his 1908 address published as *Space and Time*[1], Minkowski remarks:

> *"The whole universe is seen to resolve itself into similar worldlines, and I would fain anticipate myself by saying that in my opinion physical laws might find their most perfect expression as reciprocal relations between these worldlines."*

He derives the concepts of worldpoints and worldlines, velocity, acceleration, force, and momentum vectors, and an expression for kinetic energy in spacetime, but stops there. Not long after that, in 1914, Buckingham published his famous π-theorem, the first real treatise on the subject of dimensional analysis.[2] This was followed by Bridgman's 1922 book *Dimensional Analysis*.[3] Both Buckingham and Bridgman treated the subject in a purely algebraic fashion. In 1950 Corrsin provided a geometric proof of Buckingham's π-theorem by defining a k-dimensional cartesian space whose coordinate axes were graduated as the exponents of the k physical dimensions.[4] And in 1998 Szirtes published *Applied Dimensional Analysis and Modeling* in which he develops dimensional analysis to an art form, modeling the likes of barges pulled by tugboats, the pitch of a kettle drum, the existence criteria for black holes, and even the wavefront of a nuclear blast, using matrix methods.[5]

It is the aim of this paper to examine how *all* physical law is related to Minkowski's worldlines, and to introduce a summary of those relationships in a periodic table of dimensions. To do this, spacetime is extruded into a dimensional space $\mathbb{D}^n$, where n is the number of dimensional components in the system. The periodic table of dimensions is a system of four components: time t, space d, mass m, and electric charge q. Within this $\mathbb{D}^4$ system every physical measurement that can be derived from integer ratios of time, space, mass, and electric charge can be represented as a position vector (t, d, m, q); natural law takes its place as linear relations between these vectors. This is the space of Buckingham's π-theorem and Bridgman's dimensional analysis.

What can this do for physics teachers and students? A periodic table of dimensions can provide a common framework for teachers and students. It provides an ordered summary of every possible subject the student may encounter in a physics curriculum. As students move on from class to class, and year to year, it's helpful to have a common theme to refer back to. It provides a graphical representation of the physical relationships between dimensions. It allows us to discuss properties among the dimensions such as kinematic properties, dynamic properties, and electromagnetic properties. For teachers with an appreciation for dimensional analysis, it provides a tool that simplifies dimensional analysis to linear relations. And for engineering classes it provides a catalog of dimensional relations in terms of four fundamental dimensions, and a mathematical basis for the dimensional analysis of complex physical phenomena.

II. DIMENSIONAL COORDINATE SYSTEMS

The International System of Units SI provides a way of expressing every physical measurement known to us with its seven base units: the second s for time, the meter m for length, the kilogram kg for mass, the ampere A for electric current, the mole mol for amount of substance, the kelvin K for thermodynamic temperature, and the candela cd for luminous intensity. Of these seven, five are dimensionally unique: thermodynamic temperature can be expressed as energy, $\mathrm{kg{\cdot}m^2/s^2}$, in terms of the first three, and luminous intensity can be expressed as watts per steradian, $\mathrm{kg{\cdot}m^2/s^3}$, although there is more to that last measurement than meets the eye. Of the five that are dimensionally unique the mole is dimensionless, and the ampere is most commonly thought of as a flow of electric charge in coloumbs C per second, with the second being a redundant unit in that definition. In this way, the SI system can be reduced to four base units, for our purposes: the second, the meter, the kilogram, and the coloumb. Every other measurement can be expressed as products of integer powers of these base units.

Each of these units quantifies a dimension. In a dimensional space quantity is dimensionless. Thus, only the dimensional qualities are necessary in a table of dimensions. To reflect the dimensionally unique units of the SI system, four dimensions were chosen to form a basis in this space: time t, space d, mass m, and electric charge q. Scaling these axes according to Corrsin's prescription, by the exponents of the physical dimensions, populates an infinite vector space $\mathbb{D}^4$, with coordinate quadruplets (t, d, m, q). Using time $(1, 0, 0, 0)$, space $(0, 1, 0, 0)$, mass $(0, 0, 1, 0)$, and electric charge $(0, 0, 0, 1)$ as unit vectors forms an orthonormal basis within which all physical law can be expressed as linear relations using only integer components. Corrsin designates this space as $\mathbb{D}$-space. Any quantity expressible in SI units is expressible in this space as a four-vector: natural law being the relations among those vectors.

Every dimension, both mechanical and electromagnetic, and every dimensional constant in human literature can only populate a finite portion of an infinite $\mathbb{D}$-space. This finite portion of $\mathbb{D}$-space is what is referred to as the periodic table of dimensions. It is shown in two parts in Figs. 1 and 2: the mechanical hyperplane, and the electromagnetic hyperplanes, respectively. A full color version showing the relationship of the subsets to each other is available online.[6] A number of dimensionally rich resources including the *CRC Handbook of Chemistry and Physics* and Szirtes' *Applied Dimensional Analysis and Modeling* were surveyed for measurement units.[5,7–17] The review provided 908 unique citations of measures with clearly defined dimensionality to begin populating the table.

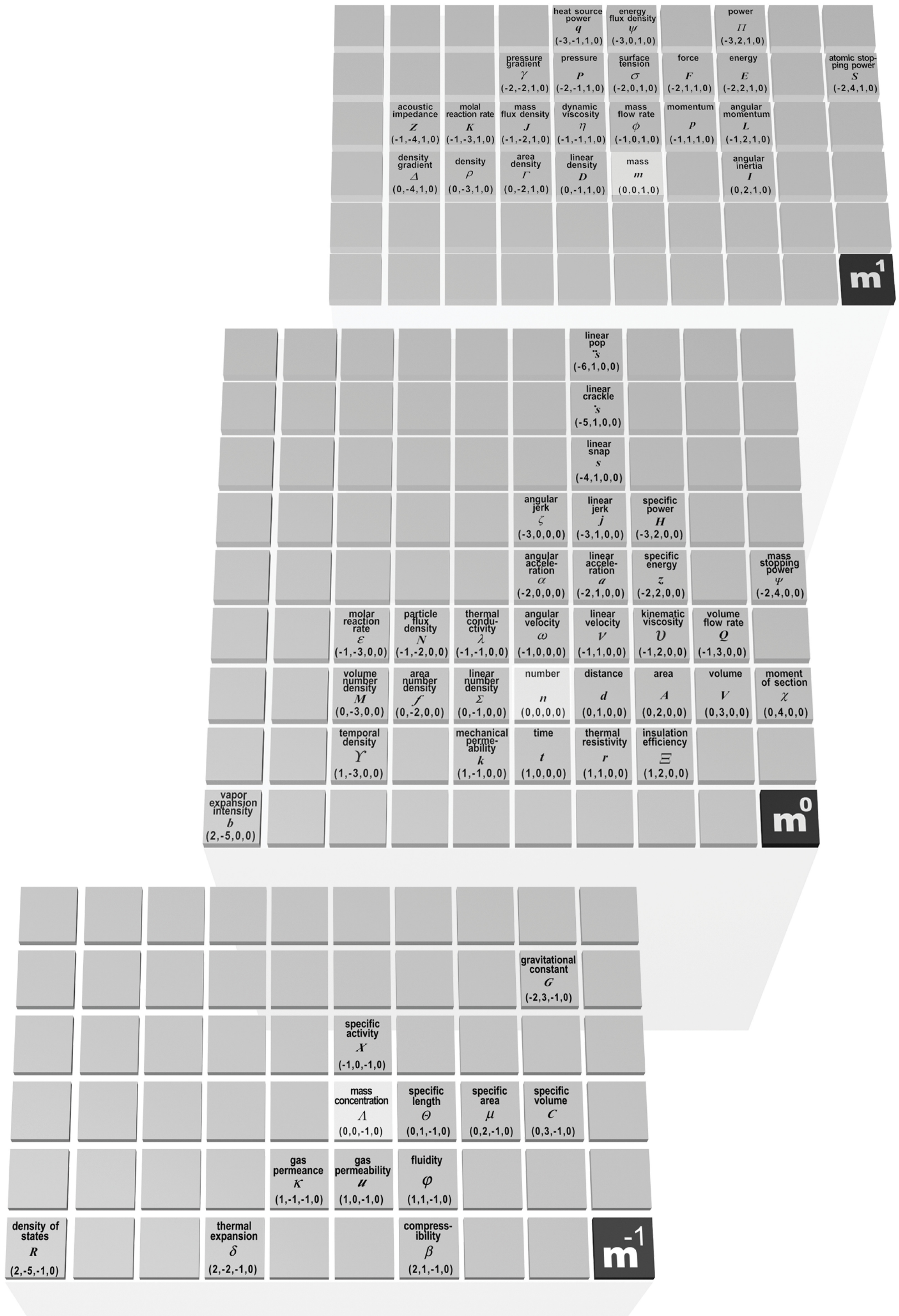
All measurements were then resolved to (t, d, m, q) components. Redundancies in the cited dimensionalities, i.e., gallons and liters are both $(0, 3, 0, 0)$, reduced this number to 105 uniquely identified dimensions in the periodic table of dimensions. A fully referenced *Catalog of Synonymous Dimensions* identified in the review is available online.[18] Among the 908 dimensions there are four unique pairs noted by Szirtes[5] (p. 42) as expressing distinctly different physical qualities yet having the same dimensionality: energy and torque, pressure and modulus of elasticity, growth of mass per unit length and dynamic viscosity, and growth of mass per unit area and mass flow per unit cross section. (Note: Szirtes does mention a fifth pair, deceleration of tree limb thickening with age and mechanical stress, but the former is not among the verified 908 dimensions.) Some of these are more arguably similar than others. For instance, modulus of elasticity is stress divided by a dimensionless strain, and the physical quality of a stress is not so far separated from pressure if one thinks of it as "pressure in a solid." Energy and torque, however, do stand out as distinctly different: like dimensional isotopes, with differing qualities in the *same place* in the table.

Organizing dimensions in this manner highlights some interesting inconsistencies in nomenclature as shown in table I. In a simple example, the term *specific* is commonly used to refer to either a quantity divided by mass, *or* a quantity divided by a dimensionally synonymous reference quantity. A more complex example is permeance and permeability: magnetic and mechanical permeance are the *product* of magnetic or mechanical permeability and distance respectively, while gas permeance is the *quotient* of gas permeability and distance. A similar inconsistency occurs when resistance and resistivity, reluctance and reluctivity, and conductance and conductivity are defined: resistivity and reluctivity are the respective products of resistance or reluctance and distance, while conductivity is the quotient of conductance and distance.

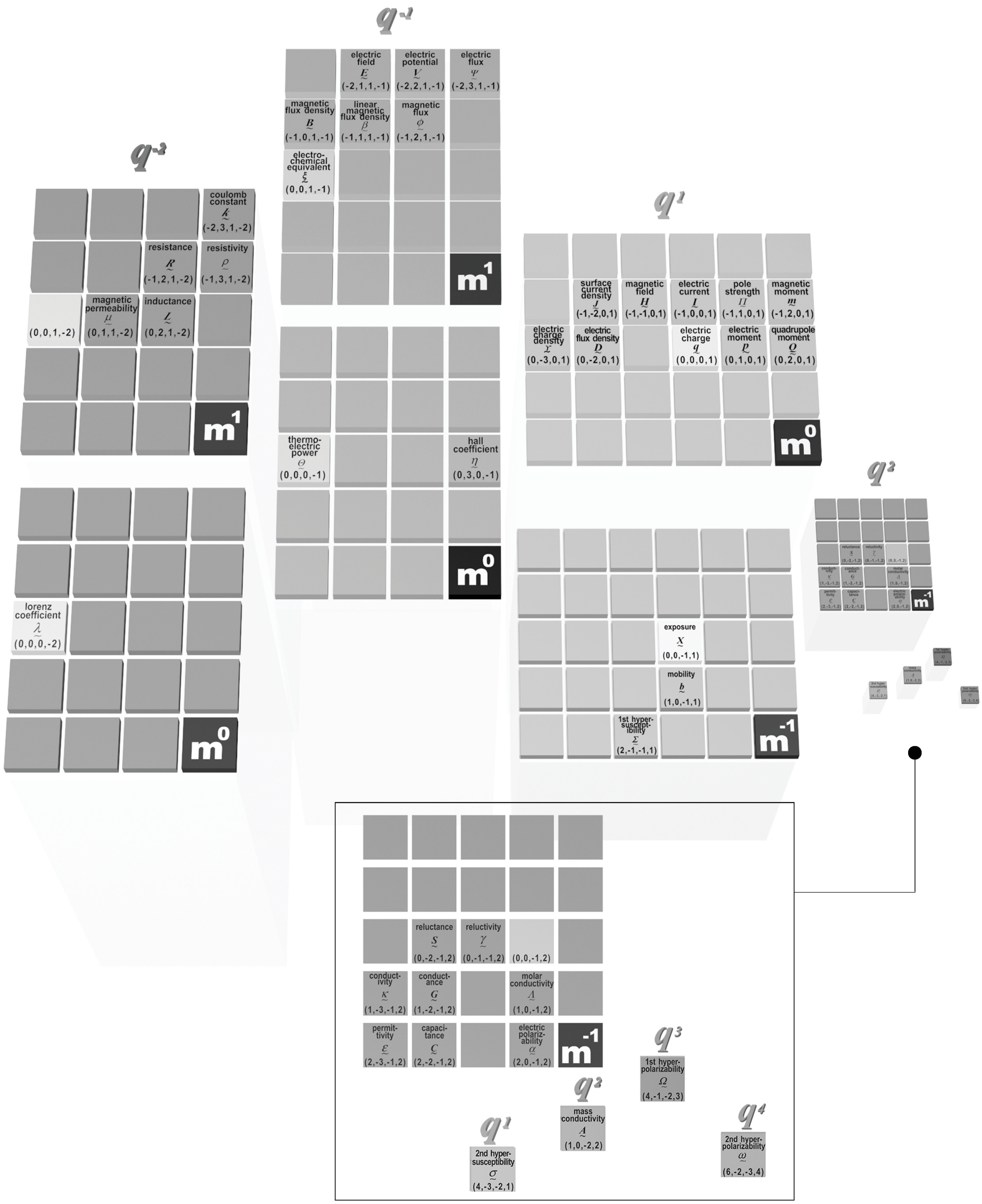
The terms flux and flux density for particles, mass, energy, and magnetism, are used consistently, with flux being defined as the surface integral (dimensionally equivalent to the product) of flux density and area. However, an ambiguity arises when we speak of electric flux and electric flux density; electric flux density was exclusively defined with dimensions $(0, -2, 0, 1)$ in this survey.[5,7,8,11,19] This is consistent with electric flux being synonymous with electric charge, i.e., electric flux density refers to the surface density of electric charge. In fact, three of four sources define electric flux as synonymous with electric charge.[5,8,11] The fourth source, Wikipedia, contains an article titled *Electric Flux*, citing Purcell and Morin, pp. 22-26, which gives the quantity discussed as the surface integral of the electric field.[19,20] Further investigation shows that Purcell and Morin never actually refer to this quantity as electric flux, but rather use the generic term flux for the surface integral of the electric field.

FIG. 1: (next page) The periodic table of dimensions: mechanical hyperplane. Three spacetime coordinate planes graduated from top to bottom as the m^1, m^0, and m^{-1} planes, respectively, give an ordered presentation of every measurement that can be derived from the base dimensions of time t, space d, and mass m. (Color online)

THE PERIODIC TABLE OF DIMENSIONS:
MECHANICAL HYPERPLANE



THE PERIODIC TABLE OF DIMENSIONS:
ELECTROMAGNETIC HYPERPLANES



So what is the surface integral of the electric field, if not electric flux? The surface integral of the magnetic field does not give magnetic flux. Instead, magnetic flux is given by the surface integral of magnetic flux density as noted above. Further investigation reveals that the product of conductance and the electric field gives the magnetic field, the product of conductance and magnetic flux density gives electric flux density, and the product of conductance and magnetic flux gives electric flux as defined by Szirtes, Allen, and Cohen.[5,8,11] Consistent with this last finding the surface integral of the electric field might be identified as electric pole strength, such that the product of conductance and electric pole strength would give (magnetic) pole strength $(-1, 1, 0, 1)$. However, this definition is not without its contradictions as well.

Physical constants are also natural to this space. When the Plank units are assigned the appropriate components of these vectors as exponents, the magnitudes of the derived dimensions *are* the physical constants with that dimensionality. Beginning with the Plank time, length, mass, and charge respectively, in SI units:

$$t_P = \sqrt{\frac{\hbar G}{c^5}} = 5.391 \times 10^{-44} \text{ s}, \tag{1}$$

$$d_P = \sqrt{\frac{\hbar G}{c^3}} = 1.616 \times 10^{-35} \text{ m}, \tag{2}$$

$$m_P = \sqrt{\frac{\hbar c}{G}} = 2.176 \times 10^{-8} \text{ kg}, \tag{3}$$

and

$$q_P = \sqrt{4\pi\epsilon_0 \hbar c} = 1.876 \times 10^{-18} \text{ C}. \tag{4}$$

The speed of light has dimensions dt^{-1} with components $(-1, 1, 0, 0)$: the Plank length divided by the Plank time gives the speed of light $c = 2.998 \times 10^8$ m/s. Angular momentum

FIG. 2: (previous page) The periodic table of dimensions: electromagnetic hyperplanes. Eleven spacetime coordinate planes graduated from top to bottom as the m^1 through m^{-3} planes, respectively, give an ordered presentation of every electromagnetic measurement that can be derived from the four base dimensions of time t, space d, mass m, and electric charge q. (Color online)

TABLE I: Inconsistent nomenclature

Initial	×	Constant	=	Result
magnetic permeability (0,1,1,-2)		distance (0,1,0,0)		magnetic permeance (0,2,1,-2) [inductance]
mechanical permeability (1,-1,0,0)		distance (0,1,0,0)		mechanical permeance (1,0,0,0) [time]
gas permeance (1,-1,-1,0)		distance (0,1,0,0)		*gas permeability* (1,0,-1,0)
resistance (-1,2,1,-2)		distance (0,1,0,0)		resistivity (-1,3,1,-2)
reluctance (0,-2,-1,2)		distance (0,1,0,0)		reluctivity (0,-1,-1,2)
conductivity (1,-3,-1,2)		distance (0,1,0,0)		*conductance* (1,-2,-1,2)
particle flux density (-1,-2,0,0)		area (0,2,0,0)		particle flux (-1,0,0,0) [angular velocity]
mass flux density (-1,-2,1,0)		area (0,2,0,0)		mass flux (-1,0,1,0) [mass flow rate]
energy flux density (-3,0,1,0)		area (0,2,0,0)		energy flux (-3,2,1,0) [power]
magnetic flux density (-1,0,1,-1)		area (0,2,0,0)		magnetic flux (-1,2,1,-1)
electric flux density (0,-2,0,1)		(undefined) (-2,5,1,-2)		*electric flux* (-2,2,1,-1)

with components $(-1, 2, 1, 0)$ and magnitude $t_P^{-1} d_P^2 m_P$, gives Plank's reduced constant $\hbar = 1.055 \times 10^{-34}$ J·s . The Coulomb constant and the gravitational constant, among others, also share this property.

To display $\mathbb{D}^4$ in 2D it was decided to use a representation of the q-axis as a separate set of tables similar to that of the lanthanides and actinides in most modern periodic tables of the elements. This allows for convenient display of the mechanical hyperplane, the electromagnetic hyperplanes, or the entire table. While names are presented in this version of the table to emphasize the properties among certain subsets of dimensions, this will not always be practical due to space constraints. To address this, commonly used variables were assigned as symbols for each dimension whenever practical. However, many ambiguities arose, as in the case of angular inertia and electric current, both I. When this occurred alternate symbols and formats were used to resolve the ambiguity. A table of the dimensions with these symbols can be found in the appendix.

III. APPLICATION IN THE CLASSROOM

I begin my own application of the periodic table of dimensions as a basis for a lifelong curriculum in math and science. The simplest concepts begin at the origin. We learn how to count: we learn how to tell time: we learn how far things are. Most of us figure out speed, before we figure out inertia. Dimensionless numbers: counting, arithmetic, angles, dozens and moles, all exist at the origin with the dimension of number. What time is it? That question is answered right next to the origin with the dimension of time. Progressing down the space axis in a positive direction from the origin we encounter geometry: lines, plane figures, solids. We also encounter the physical manifestation of these dimensions: distances, surface areas, and volumes. And then we're ready for our first physics class.

Integrating distance measurements into our curriculum from the negative t direction we discover kinematics: acceleration, velocity, displacement,... This same integration with respect to the origin reveals the parallel kinematics of angular motion. With the introduction of inertia along the mass axis, dynamics takes shape: momentum, force, energy, and some angular counterparts, angular inertia, and angular momentum. I find it very useful to have a poster of this periodic table to draw a visual picture of these relationships for my students. I've even constructed a 3D version, shown in Fig. 3, using 3D-printed plastic spheres floating on magnetic fields.

If we stop to look at the relationships these dimensions have to each other, we begin to notice groupings within the table. There's a geometric grouping of distance, area, and volume along the positive d-axis, and density groupings along the negative d-axes in both the spacetime plane and the m^1-plane. The dimensions of linear kinematics, reported all the way out to the sixth time-derivative of motion, show up in the d^1 group. Parallel to those, in the d^0 group along the negative t-axis, we find a grouping of our rotational kinematics dimensions,

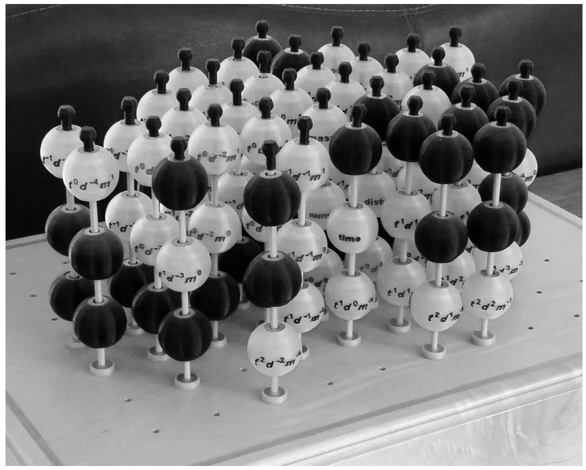


FIG. 3: A 3D adaptation of a previous version of the periodic table of dimensions with white spheres representing known dimensions, and black spheres representing the unknown dimensions. The spheres are 3D-printed plastic, floating on magnets.

recorded to the third time-derivative. The dimensions of dynamics group in quadrant II of the m^1-plane, all the mechanical dimensions group in the mechanical hyperplane $(t, d, m, 0)$, and all electrical dimensions group along the q-axis, intersecting the mechanical hyperplane strongest in the dynamics grouping.

Students get a great kick out of the higher time-derivatives of motion: snap, crackle, and pop. The obvious association with a breakfast cereal gives them an insight into a physicist's sense of humor. Because of this, one problem I love assigning is to derive an equation of motion beginning with constant pop, the sixth time-derivative of motion. The repetitive integrations produce a series of factorial denominators that most students find eye opening. The connection of the physical to the abstract is a powerful example of a use for all that math we made them learn. The quadratic equation of motion for constant acceleration also provides a great opportunity for this as an application of the quadratic formula in lower level courses. These are the moments I see the bulbs lighting up in my students' heads.

General periodic trends occur along each of the four axes. A trend from static (time) to dynamic (velocity, acceleration, jerk,...) occurs parallel to the t-axis moving from positive to negative. Parallel to the d-axis, in a positive to negative direction, we observe a trend from integrating (volume, area, distance) to differentiating (...density, density gradient). From positive values along the m-axis toward negative values, the properties proceed from convergent, to divergent, e.g., density to permeability and thermal expansion. And finally, from positive to negative along the q-axis, we observe a trend from conductance to resistance. Each of these trends displays its strongest attributes closest to the axis running through the origin of the table, and weakening as the distance from that axis increases.

Other properties apparent among the dimensions may, or may not have regular trends. Independent dimensions are characterized as independent vectors in this space. Then, the number of independent dimensions obviously increases the further from the origin one goes, but there is also a definite pattern, dominated by the primes as shown in Fig. 4, as to which dimensions are independent. The independence of dimensions has a conspicuous effect on the distribution of measurements recorded in the table, showing a strong preference for the negative t and positive m directions. Additionally, of the 908 measurements surveyed (resolved to 105 dimensions), not one dimension has more than two negative components, while what we actually measure seems limited to three, e.g.: g-factor in SI units kg·s^{-1}·C^{-1}·T^{-1}, with dimensions (0,0,0,0); specific molar heat capacity J·kg^{-1}·mol^{-1}·K^{-1}, (0,0,-1,0); and mechanical permeability kg·m^{-2}·s^{-1}·Pa^{-1}, (1,-1,0,0). Some notable examples of dependencies are: dynamic viscosity (-1,-1,1,0) and fluidity (1,1,-1,0); pressure (-2,-1,1,0) and compressibility(2,1,-1,0); and linear velocity (-1,1,0,0) and specific energy (-2,2,0,0).

It makes some sense that we would not need to measure reciprocal relationships or multiples of things we already measure, but there are also some notable "holes" in the table. For instance, the lack of any measurement with components $(0, 1, 1, 0)$. No scalar multiples of that vector, other than the trivial solution, appear anywhere in the table. One would think that in all of the *CRC Handbook of Chemistry and Physics* and Szirtes' 790 page exposition on dimensional analysis and modeling, among other resources, there would be at least one mention of a measurement with dimensions md. It is also striking that not one measurement of dimensions md^nt^1 or md^nt^2 appears in the literature, and the only non-zero scalar multiples of those are specific activity $m^{-1}t^{-1}$ and the gravitational constant $m^{-1}d^3t^{-2}$. The survey undertaken here was in no way meant to be exhaustive, however, it seems highly unlikely that a dozen or so measurements all with dimensions md^nt^1 or md^nt^2, and their scalar multiples, were consistently omitted from the reputable sources surveyed.

The most significant relationship among the dimensions in $\mathbb{D}$-space is that of natural law. In the periodic table of dimensions, natural law takes the form of linear algebra.

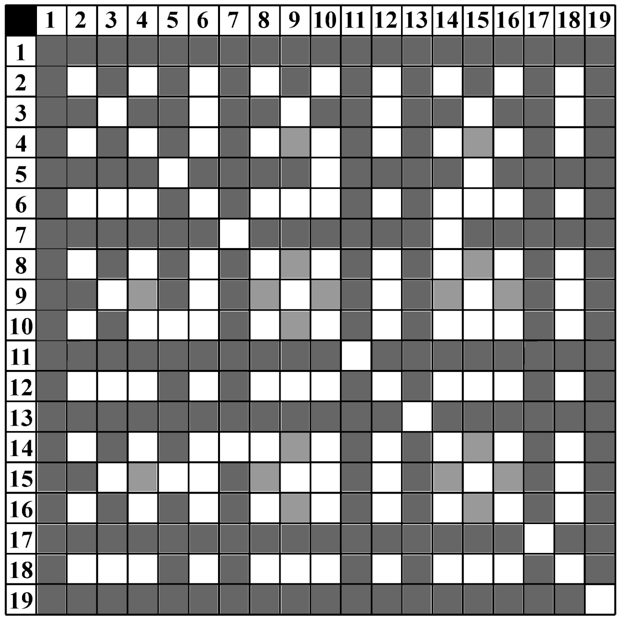


FIG. 4: Independent dimensions in the periodic table. The first quadrant of the m^0 spacetime plane in $\mathbb{D}$-space. The black square is the origin. The dark gray squares show the independent dimensions in the plane that are formed by the prime numbers; light gray squares show independent dimensions resulting from compound numbers. The white squares are the dependent vectors.

The components of any dimension are the components of its position vector in $\mathbb{D}$-space. Exponents on dimensions become scalar multiples of that vector. Every product in natural law can be expressed as vector addition and scalar multiplication; all terms in any expression of natural law must be dimensionally homogeneous because only like terms can be combined. Since all numerical constants have components $(0, 0, 0, 0)$, their value cannot be determined by dimensional analysis alone, but the power of this method for finding scalable relations should not be dismissed. The following examples demonstrate this method for Newton's second law, the ideal gas law, the equation for the speed of light in a vacuum, and the quadratic equation of linear motion for constant acceleration.

Newton's second law $F = ma$ presents a simple and straightforward example of this method:

$$\begin{array}{ccccc} F & = & m & & a \\ \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} & = & \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} & + & \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \end{array}. \tag{5}$$

Not only does the table provide an algebraic interpretation for this calculation, but also a geometric one. Students can follow this equation as we "move" through the table from mass at $(0, 0, 1, 0)$, one unit out the positive d-axis and 2 units in the negative t direction to force.

The ideal gas law $PV = nRT$ provides a look at what appears to be a dimensional constant. However, upon closer examination, one finds that while R is not exactly *unitless*, the dimensions of its units, i.e., $J/(mol \cdot K)$ cancel such that it is a dimensionless constant. Therefore, this equation takes the form

$$\begin{array}{ccccccccc} P & & V & = & n & & R & & T \\ \begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \end{bmatrix} & + & \begin{bmatrix} 0 \\ 3 \\ 0 \\ 0 \end{bmatrix} & = & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} & + & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} & + & \begin{bmatrix} -2 \\ 2 \\ 1 \\ 0 \end{bmatrix} \end{array}. \tag{6}$$

Geometrically this can be seen as a movement from pressure, at $(-2, -1, 1, 0)$, three units in the positive d direction to arrive at energy which is dimensionally synonymous with temperature.

The equation for the speed of light in a vacuum,

$$c = \frac{1}{(\varepsilon_0 \mu_0)^{\frac{1}{2}}}, \tag{7}$$

gives us the opportunity to see how an exponent behaves in the arithmetic of $\mathbb{D}$-space. Since it appears in the denominator, the exponent will introduce subtraction to our vector equation, and the constant in the numerator is dimensionless with components $(0, 0, 0, 0)$

such that the equation takes the form

$$\begin{array}{ccccccc} c & = & 1 & \cdot & (\varepsilon & & \mu)^{-\frac{1}{2}} \\ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} & = & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} & -\frac{1}{2} & \left(\begin{bmatrix} 2 \\ -3 \\ -1 \\ 2 \end{bmatrix} \right. & + & \left. \begin{bmatrix} 0 \\ 1 \\ 1 \\ -2 \end{bmatrix} \right) \end{array}. \tag{8}$$

From a geometric perspective, beginning at permittivity $(2, -3, -1, 2)$, we slide 2 units "down" the q-axis, and then move 1 unit in both the positive d direction and the positive m direction to arrive at the unnamed element $(2, -2, 0, 0)$. From this position, multiplying by negative one-half moves us half-way to the origin and then reflects us across it to the velocity element $(-1, 1, 0, 0)$.

The quadratic equation of motion for constant acceleration is an example of how an equation with multiple terms behaves in this space. Since we will have three terms, each term must have identical dimensions if they are to be added together in $\mathbb{R}$-space. Not only that, but the principle of dimensional homogeneity requires that the dimensionality on both sides of the equal sign be the same, such that every term on one side must have the same dimensions as any terms on the other. Our vector equation then becomes

$$\begin{array}{ccccccccccccc} d_f & = & d_i & + & v_i & \cdot & t & + & \frac{1}{2} & \cdot & a & \cdot & t^2 \\ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} & = & \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} & ; & \left(\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right. & + & \left. \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right) & ; & \left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right. & + & \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} & +2 & \left. \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right) \end{array}. \tag{9}$$

In a geometric sense each term moves us through the table to our destination, in this case $(0, 1, 0, 0)$. Every journey must arrive at the same destination. Then, and only then, can the values of each term be combined in $\mathbb{R}$-space.

For engineering students the periodic table of dimensions, and *The Catalog of Synonymous Dimensions*, provide a reference source for the application of π-theorem to their physical modeling activities. Szirtes has developed this to an art form, so I will refer you to his text[5] for the theory and only give a brief overview of its application as it relates to our discussion here. The goal is to derive the dimensionless relations among the variable physical quantities involved in any particular problem. With those relationships determined, a dimensionally homogeneous equation can be found such that scale factors for the variable quantities at play can be established for modeling the phenomenon in question.

To do this, Szirtes develops the *dimensional set*, an augmentation of the dimensional matrix derived from the relations of interest. This augmented matrix is then partitioned into 4 parts called the **A**, **B**, **C**, and **D** matrices as shown in Fig. 5. The **A** matrix is a square $N \times N$ matrix, where N equals the number of fundamental dimensions involved. It consists of the independent variable quantities contributing to the dimensionless products formed using π-theorem. The **B** matrix is formed from the dependent variables we wish to determine dimensional relationships for among the independent variables. We will be able to determine as many dimensionless relations, π_i's, as there are dependent variables. The **D** matrix is a $D \times D$ identity matrix, where D is the number of dependent variables. And the **C** matrix is found by the matrix equation $\mathbf{C} = -(\mathbf{A}^{-1} \cdot \mathbf{B})^{\mathrm{T}}$.

I'll use Szirtes example 18-35, *velocity of Collapse of a Row of Dominoes*, to demonstrate the method. The variables in play are the velocity v, their separation λ, thickness δ, height h, and the acceleration of gravity g; the fundamental dimensions are time t and distance d. With only two fundamental dimensions in our problem, the **A** matrix will be a 2×2 matrix and there will be three dimensionless products (five variables - two dimensions) such that the **D** matrix will be a 3×3 identity matrix. From this information we can set up our dimensional set:

$$\begin{array}{c} \\ d \\ t \\ \pi_1 \\ \pi_2 \\ \pi_3 \end{array}
\begin{array}{c} \begin{array}{ccc|cc} v & \lambda & \delta & h & g \end{array} \\
\left[\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ -1 & 0 & 0 & 0 & -2 \\ \hline 1 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right] \end{array} , \tag{10}$$

where π_1, π_2, and π_3 give us the exponents of the dimensionless products such that

$$\pi_1 = \frac{v}{\sqrt{hg}} \ ; \quad \pi_2 = \frac{\lambda}{h} \ ; \quad \pi_3 = \frac{\delta}{h} \ . \tag{11}$$

According to π-theorem any one of these dimensionless products is equal to some arbitrary

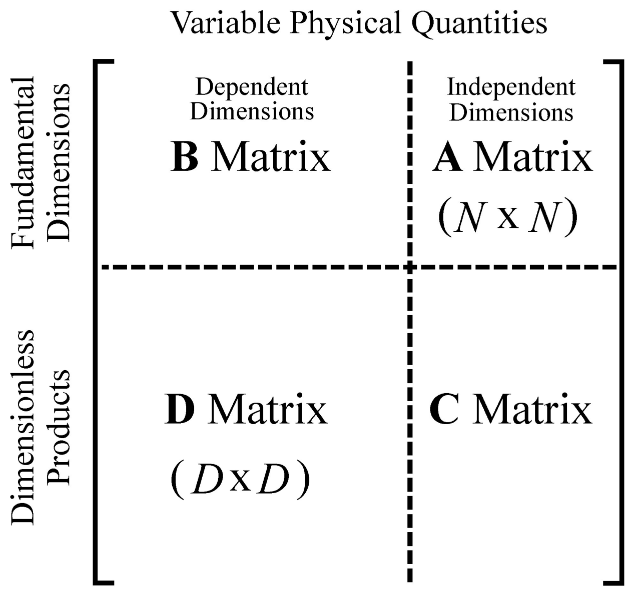


FIG. 5: The dimensional set. An augmentation of the dimensional matrix derived from the quantities of interest and their fundamental dimensions. The augmented matrix is partitioned into 4 parts: the **A**, **B**, **C**, and **D** matrices. The **A** matrix is a square matrix of the dimensions considered independent in the problem. The **B** matrix is formed from the remaining, so called, dependent dimensions. The **D** matrix is an identity matrix that will give us each dependent variable's relation to a dimensionless product, and the **C** matrix which gives the remainder of that dimensional product.

function Ψ of the other remaining dimensionless products so that

$$\frac{v}{\sqrt{hg}} = \Psi(\pi_2, \pi_3), \tag{12}$$

which can be solved for v as

$$v = \sqrt{hg} \cdot \Psi(\pi_2, \pi_3). \tag{13}$$

The arbitrary function Ψ must be found by experimentation; a task perfectly suited to a physics lab on dimensional analysis.

However, the point of this exercise was to demonstrate the utility of the periodic table of dimensions and *The Catalog of Synonymous Dimensions*. This problem was intentionally chosen to be simple, but if we were analyzing, for instance, a barge being pulled by a tug-boat, the pitch of a kettle drum, or the wavefront of a nuclear blast, you can imagine how much more difficult it could be to determine the dimensionality of every variable involved. Since every discipline tends to run across the same variables repeatedly, periodic tables of dimensions can be customized for different disciplines, and by thinking in terms of synonymous dimensions we can also narrow the field in those cases where a simple table is lacking, making the periodic table of dimensions and its associated catalog of dimensions a handy resource for quick reference.

IV. CONCLUSION

The realization of Minkowski's anticipation is almost complete. A periodic table of dimensions certainly summarizes all of natural law. We have seen how it organizes the dimensional elements of physics, how it reveals the organization of their properties, the disorganization of our nomenclature, and even how it can help organize our own thought process as we teach physics. We can use it to direct a curriculum, to summarize the elements of physics and the relation of their properties. We can use it to examine natural law both geometrically and algebraically, and we can use it to aid us in the understanding of complex physical phenomena. It's enough to make a chemistry teacher jealous. However, the question still remains: are Minkowski's worldlines and the dimensional position vectors introduced by Corrsin really the same construct from a transformed perspective?

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Appendix: Dimensional Listing

(a) Dimensional listing: the spacetime plane

Spacetime plane		
Dims	Dimension	Symbol
(-6,1,0,0)	linear pop	s
(-5,1,0,0)	linear crackle	s
(-4,1,0,0)	linear snap	s
(-3,0,0,0)	angular jerk	ζ
(-3,1,0,0)	linear jerk	j
(-3,2,0,0)	specific power	H
(-2,0,0,0)	angular acceleration	α
(-2,1,0,0)	linear acceleration	a
(-2,2,0,0)	specific energy	z
(-2,4,0,0)	mass stopping power	Ψ
(-1,-3,0,0)	molar reaction rate	ε
(-1,-2,0,0)	particle flux density	N
(-1,-1,0,0)	thermal conductivity	λ
(-1,0,0,0)	angular velocity	ω
(-1,1,0,0)	linear velocity	v
(-1,2,0,0)	kinematic viscosity	υ
(-1,3,0,0)	volume flow rate	Q
(0,-3,0,0)	volume number density	M
(0,-2,0,0)	area number density	f
(0,-1,0,0)	linear number density	Σ
(0,0,0,0)	number	n
(0,1,0,0)	distance	d
(0,2,0,0)	area	A
(0,3,0,0)	volume	V
(0,4,0,0)	moment of section	χ
(1,-3,0,0)	temporal density	Υ
(1,-1,0,0)	mechanical permeability	k
(1,0,0,0)	time	t
(1,1,0,0)	thermal resistivity	r
(1,2,0,0)	insulation efficiency	Ξ
(2,-5,0,0)	vapor expansion intensity	b

(b) Dimensional listing: mass axis

Mass axis		
Dims	Dimension	Symbol
(-3,-1,1,0)	heat source power	q
(-3,0,1,0)	energy flux density	ψ
(-3,2,1,0)	power	Π
(-2,-2,1,0)	pressure gradient	γ
(-2,-1,1,0)	pressure	P
(-2,0,1,0)	surface tension	σ
(-2,1,1,0)	force	F
(-2,2,1,0)	energy	E
(-2,3,-1,0)	gravitational constant	G
(-2,4,1,0)	atomic stopping power	S
(-1,-4,1,0)	acoustic impedance	Z
(-1,-3,1,0)	molal reaction rate	K
(-1,-2,1,0)	mass flux density	J
(-1,-1,1,0)	dynamic viscosity	η
(-1,0,-1,0)	specific activity	X
(-1,0,1,0)	mass flow rate	Φ
(-1,1,1,0)	momentum	p
(-1,2,1,0)	angular momentum	L
(0,-4,1,0)	density gradient	Δ
(0,-3,1,0)	density	ρ
(0,-2,1,0)	area density	Γ
(0,-1,1,0)	linear density	D
(0,0,-1,0)	mass concentration	Λ
(0,0,1,0)	mass	m
(0,1,-1,0)	specific length	Θ
(0,2,-1,0)	specific area	μ
(0,2,1,0)	angular inertia	I
(0,3,-1,0)	specific volume	C
(1,-1,-1,0)	gas permeance	κ
(1,0,-1,0)	gas permeability	u
(1,1,-1,0)	fluidity	φ
(2,-5,-1,0)	density of states	R
(2,-2,-1,0)	thermal expansion	δ
(2,1,-1,0)	compressibility	β

(c) Dimensional listing: electromagnetic axis

Charge axis		
Dims	Dimension	Symbol
(-2,1,1,-1)	electric field	$\underset{\sim}{E}$
(-2,2,1,-1)	electric potential	$\underset{\sim}{V}$
(-2,3,1,-2)	coulomb constant	$\underset{\sim}{k}$
(-2,3,1,-1)	electric flux	$\underset{\sim}{\Psi}$
(-1,-2,0,1)	surface current density	$\underset{\sim}{J}$
(-1,-1,0,1)	magnetic field	$\underset{\sim}{H}$
(-1,0,0,1)	electric current	$\underset{\sim}{I}$
(-1,0,1,-1)	magnetic flux density	$\underset{\sim}{B}$
(-1,1,0,1)	pole strength	$\underset{\sim}{\Pi}$
(-1,1,1,-1)	linear magnetic flux density	$\underset{\sim}{\beta}$
(-1,2,0,1)	magnetic moment	$\underset{\sim}{m}$
(-1,2,1,-2)	resistance	$\underset{\sim}{R}$
(-1,2,1,-1)	magnetic flux	$\underset{\sim}{\Phi}$
(-1,3,1,-2)	resistivity	$\underset{\sim}{\rho}$
(0,-3,0,1)	electric charge density	$\underset{\sim}{\Upsilon}$
(0,-2,-1,2)	reluctance	$\underset{\sim}{S}$
(0,-2,0,1)	electric flux density	$\underset{\sim}{D}$
(0,-1,-1,2)	reluctivity	$\underset{\sim}{\gamma}$
(0,0,-1,1)	exposure	$\underset{\sim}{X}$
(0,0,0,-2)	lorenz coefficient	$\underset{\sim}{\lambda}$
(0,0,0,-1)	thermoelectric power	$\underset{\sim}{\Theta}$
(0,0,0,1)	electric charge	$\underset{\sim}{q}$
(0,0,1,-1)	electrochemical equivalent	$\underset{\sim}{\xi}$
(0,1,0,1)	electric moment	$\underset{\sim}{p}$
(0,1,1,-2)	magnetic permeability	$\underset{\sim}{\mu}$
(0,2,0,1)	quadrupole moment	$\underset{\sim}{Q}$
(0,2,1,-2)	inductance	$\underset{\sim}{L}$
(0,3,0,-1)	hall coefficient	$\underset{\sim}{\eta}$
(1,-3,-1,2)	conductivity	$\underset{\sim}{\kappa}$
(1,-2,-1,2)	conductance	$\underset{\sim}{G}$
(1,0,-2,2)	mass conductivity	$\underset{\sim}{A}$
(1,0,-1,1)	mobility	$\underset{\sim}{b}$
(1,0,-1,2)	molar conductivity	$\underset{\sim}{\Lambda}$
(2,-3,-1,2)	permittivity	$\underset{\sim}{\varepsilon}$
(2,-2,-1,2)	capacitance	$\underset{\sim}{C}$
(2,-1,-1,1)	first hypersusceptibility	$\underset{\sim}{\Sigma}$
(2,0,-1,2)	electric polarizability	$\underset{\sim}{\alpha}$
(4,-3,-2,1)	second hypersusceptibility	$\underset{\sim}{\sigma}$
(4,-1,-2,3)	first hyperpolarizability	$\underset{\sim}{\Omega}$
(6,-2,-3,4)	second hyperpolarizability	$\underset{\sim}{\omega}$

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